The Mathematics Teacher

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SOME IDEAS ON THE STUDY OF GEOMETRY.

BY CHARLES R. SHULTZ.

In the preparation of this paper I have followed certain suggestions on the study of Geometry contained in the Report of the Committee of Fifteen on a Geometry Syllabus. The introduction to this report gives an historical account of various attempts to reform the teaching of geometry. This section should prove, therefore, not only interesting but extremely valuable to students and teachers of mathematics. From it we gleam some very noteworthy facts in regard to the progress that has been made in the attempts to place the study of geometry on a sane and satisfactory footing in the school curriculum.

Beginning as a practical subject, an outgrowth of primitive practice in mensuration and physical research, geometry soon proved to be a very fertile field for the study of logic and for the use of formal processes of reasoning. Under the influence of Pythagoras, the application of geometry to practical affairs was at first extended, since he associated it very closely with arithmetic. But the Platonic school despised the science of numbers; they considered the application of geometry to arithmetic and surveying as degrading and vulgar. This school, therefore, emphasized the logical aspect of the subject and did much to increase the rigor and to improve the methods employed in its demonstrations. When Plato inscribed over the entrance to his Academy the words, "Let no one who is not acquainted with

geometry enter here," he had reference, I am sure, to training in the formal processes of reasoning which this subject so beautifully exemplifies and which is a necessary preparation for the study of higher philosophy, rather than to knowledge of the principles of geometry, as geometry.

The great name, of course, in the early history of the subject is that of Euclid, the greatest systematizer of his age, perhaps of any age, who collected and organized all that was then known of geometry into an almost perfect piece of logic. A most remarkable fact of mathematical history is that this body of knowledge as devised by him for adult students over two thousand years ago should have persisted almost unchanged through all these centuries, and be taught at the present time to boys and girls of fifteen or sixeen years of age, as part of their preparation for the general activities of life. It seems to me that we should remember that geometry was originally designed for adults, when we find, as we too often do, that a large number of our students fail to make satisfactory progress in the study of geometry as generally taught. For we require of our children that they be able to give rigorous demonstrations and discover original proofs of abstract propositions, when the keen-minded, speculative Greeks, of mature age, found the same tasks laborious, and the royal Ptolemy asked for "an easier road to geometry."

Another point of interest to me as a teacher of geometry is the attitude which leading mathematicians have, from time to time, taken toward the "Elements" of Euclid. Cajori describes it as the "ever-recurring 'see-saw' between the strictly logical presentation as represented by Euclid, and the more intuitive form which makes greater use of concrete material." It seems to me that it is this latter idea which has gained greatest favor and is to-day being generally accepted.

In France, the attitude almost from the beginning was against Euclid, and after a century or more of criticism and experiment, a great text on "Elementary Geometry" was written by Legendre. Its style was more attractive than that of Euclid, and it placed more emphasis on intuition and less on logic; it contained more matter on solid geometry, and had many points of fusion with arithmetic and trigonometry. This book became the lead-

ing text in France, and had great influence in shaping the early courses in geometry in America. The French have continued to advance in the direction just indicated, and in recent texts and official courses of study, stress is laid upon the use of motion in geometry and upon practice in geometrical drawing; the practical is emphasized wherever possible, and intuition is more fully recognized than ever before; the division of the subject into plane and solid is not adhered to, and the laboratory method —an English idea—is being experimented with. If there is anything in the experience of the French by which we could profit in our work, it is, I should say, in requiring less rigor and in giving freer play to intuition. When a pupil says "I can see that is true without proving it," he should more frequently be given credit for his insight rather than be confused by hair-splitting logical distinctions. I am fully convinced that we often cause a proposition to become hazy in the mind of a pupil and make the whole subject difficult and vague to him by requiring a formal proof when a fact is fully appreciated without it. Other points which we might give more attention to are greater attention to the practical, to the fusion of certain parts of plane and solid geometry, and to the application of algebra and trigonometry when it can be conveniently and profitably done.

In Germany, during the nineteenth century, the influence of Pestalozzi and Herbart made itself felt and gave rise to the genetic, or so-called "heuristic" method of presenting the subject. This is considered the natural method of presentation, since it gives the pupil freedom to express himself in his own way and allows him to discover for himself geometrical truths. The movement toward making geometric instruction genetic is supported by Felix Klein, president of the International Commission on the Teaching of Mathematics, and one of the greatest of living mathematicians, and it is being generally adopted in the schools of Germany. Some study of this method, its aims and its results, is not out of place, therefore, in a paper of this kind.

Herbart gives us the principle that "the pupil must know from the beginning what is aimed at, if he is to employ his whole energy in the effort of learning." In many of the demonstrations of the present-day texts, the opposite principle seems to

have been employed, for the pupil is carefully kept in the dark as to the purpose of the method of procedure until the conclusion is suddenly reached. A fine example of this artificial method may be seen in the usual proof of the theorem concerning the square of the side opposite an acute angle of a triangle. When teaching this theorem, I always find it necessary to invert the order of the steps, beginning with the square of the opposite side, and make such substitutions as we find it possible to make from the relations existing between the various lines until we derive the desired result. Through this procedure I find the pupil able to discover the purpose of each step, and I believe it appeals to him as a reasonable process. A pupil should also, it seems to me, be let into the secret as to the reason for drawing an auxiliary line in a certain way. Does not this hidden plan, so frequently employed, encourage, yes even compel, a pupil to memorize his lesson? Is it not better pedagogy and will the results not be at least as good if with the aid of leading questions we permit the pupil to discover the theorem as well as the plan of its proof for himself? To use a single illustration, when we come to the theorem concerning the lateral area of a prism, why not have the pupil see or handle a prism, discuss with him the area of each of its faces, and let him draw his own conclusion? He will thus have formulated the theorem for himself, determined the steps in the proof of it, and having done this once, he should be able to do it again. In this way the necessity of his memorizing a proof will have been obviated. He will also have discovered what previous theorems are necessary in his proof, and will consequently appreciate more fully the relation between various parts of the subject than would be possible by the more formal method. If the teacher will take part of the period to study with the class the next day's lesson, I believe the subject will be made more interesting to his pupils, and they in turn will get a more intelligent view of geometry as a whole, and derive at the same time greater power of attack in original work. One might add that the habit of requiring pupils to give in advance, and perhaps informally, a definite plan of proof will also aid in securing this result. When a pupil draws an auxiliary line, he should be able to tell what he plans to accomplish with it; if two lines are to be proved equal, he

should be able to give his plan of attack. And so in the demonstration of many theorems, he should be required to outline the steps of the proof before he begins to give it.

Another movement of considerable significance also began in Germany—that of a preliminary course in observational and constructive work in geometry. This idea is meeting with much favor in our schools, and in many places such courses are being introduced in the upper grades. The Committee of Fifteen, in its Report, recommends such a course, and there are many reasons why it should be generally adopted. Of course, this work should not be demonstrational, but based entirely upon observation, experiment, and measurement. Children are interested in work of this nature, and even in the early grades the desire to handle and construct geometrical forms is strong. Such a course may be easily carried on in connection with courses in manual training; but even elsewhere work in the construction of perpendiculars, the bisecting of lines and angles, the making of patterns containing geometrical figures, paper folding, and the use of ruler and compasses, can be profitably undertaken. The mensuration of simple forms, such as the square, cube, triangle and rectangle, may be studied quite early. In this way the pupil soon becomes familiar with the more obvious properties of various figures. Such courses would certainly enrich the work of the upper grades without adding much weight, and the knowledge thus obtained would not only be valuable for the later courses in geometry, but would add much to the preparation for any work that might follow, in school or out.

If, however, some such work is not done in the grades, it seems to me absolutely essential that enough time be taken from the regular course in geometry to acquaint the student first with the subject matter involved in the course. He should become familiar with the general facts of geometry, the nature of geometric figures, and the use of instruments for geometric constructions, before beginning work in formal demonstration. Very frequently, I believe, students—and some who are not dull students—go through the course in geometry without getting any clear notion of what an angle, or a perpendicular, really is. Clear-cut concepts of the elementary geometric forms in the minds of the pupils before formal geometry is begun will obviate

such difficulties. And concrete work in the early part of the course followed by the gradual introduction of formal logic will aid materially in vitalizing the work in geometry.

What is known as the "Perry Movement" has done much to awaken interest, in America as well as in England, in the effort to make the teaching of geometry more efficient. This reform was begun as a reaction against the extreme rigor of treatment required in English schools, and their absurd system of examinations which required that only one order of theorems be accepted, and that a large part of the geometry be memorized, even to the lettering of the figures. It placed great emphasis upon the practical applications of the subject and its utilitarian value, though it possibly went to extremes in this respect. But we owe to the Perry Reform much of the present tendency in our teaching toward closer correlation between geometry and real life; we are coming to believe now that geometry should be applied to real problems; that is, to problems taken from the pupil's actual experience or from those fields in which he is likely to work later, instead of problems that were interesting to the Greeks, or to the students of the Middle Ages, but which have ceased to appeal in the least to the children of the present. Such problems should be gathered from the pupil's every-day activities, out of situations actually encountered by him, and they should be to some extent, at least, actually formulated by him. The study of anything that cannot be put to use in some way in the actual life of an individual is really of slight educational value. To be educative, study must have motive and interest; so if the problems of algebra, geometry, arithemtic, do not appeal to the pupil as having some connection with living experience, he will have little interest in their solution.

There are many advantages, it seems to me, that come from a skillful use of such new material in our courses in mathematics, though there may be also certain disadvantages. In the gathering of real roblems, many artificial ones may appear, problems which are made to fit a certain situation but which have no real existence in actual life; too many problems of various kinds may be admitted, thus bringing confusion and perhaps obscuring the principles to be taught; again, the interest of the class may be dissipated in too much of this work by one's making it an end

in itself, and thus losing sight of the fact that a knowledge of geometry is, after all, the real goal to be sought. On the other hand, the skillful teacher will make such use of these problems as to hold the interest and enthusiasm of his pupils while he is teaching them the principles of pure mathematics. The application of geometry to many such problems will impress upon a student the fact that his mathematics can be used, and will give him knowledge and judgment of how to use it. And the practice obtained in this way will make his algebra and geometry tools which can be used when needed, and used with intelligence and efficiency. But not the least advantage of such concrete, real problems consists in the fact that they rarely "come out even," and the pupil is consequently compelled to use his knowledge of decimals, of common fractions, short methods, and methods of checking results. He will discover that results in actual practice are rarely absolutely correct but only approximate, and he must exercise his judgment as to the degree of approximation necessary in the given problem. And I shall add this advantage, that if we teach the subject in such a way that it gives contact with real, serious, everyday life, rather than spend all the time in the study of pure theory, the study of geometry will furnish increased earning power to the pupil when he leaves school, and enable him to get a better position than he otherwise could.

In the syllabus proper, submitted by the Committee of Fifteen, certain suggestions are made involving change in the manner of treatment and the elimination of some of the theorems. We recall the fact that geometry was originally devised as a study for adult students; in early modern times it was taught only in colleges and universities; and even but a century ago, a graduate of Harvard College, in discussing the course, said, "the sophomore year gave us Euclid to test our strength." If we now teach this subject four years earlier in the course than was done so recently, and to pupils at least five or six years younger, we certainly should teach it differently. The attempt to simplify the course and confine the theoretical side to essential principles is a move in the right direction. Some theorems, including those in incommensurables, require a rigorous demonstration of such nature that their real significance cannot be fully grasped by high school students. The truth of many other

theorems is obvious, and such need only an informal proof. By placing less emphasis upon these theorems, the teacher will be able to shorten the course at certain points, and supplement elsewhere with new material. This will vitalize the work and make it appeal to the interests of the average pupil, especially those who are not inclined toward abstract reasoning. We should, of course, not cease to be logical in the work of geometry, but we should cease, it seems to me, to teach the subject as pure logic. Geometry may have served well as a basis for the study of logic when taught to adult students, men of maturity of mind. But such conditions no longer exist. Should we not now, rather, put less emphasis on dead formalism and teach more earnestly the living facts of geometry, as geometry, as the study of the properties; construction and measurement of geometrical figures, and the application of these principles to real situations in life? And if we do this, we could well give more attention to geometry as an organized body of knowledge. In it are the basal theorems, those of great importance around which the whole subject clings. Others are important only because they serve to establish the relation or connection between various parts. Still others are mere corollaries, independent largely of the subject as a whole, and having little or no application in practice. The subject of geometry has been taught too much as though all theorems were on a dead level of importance. On the contrary, much stress should be laid upon these few basal theorems, such as the "equal triangle" theorems in plane geometry, and in solid geometry the one beginning, "a line is perpendicular to a plane," or the theorem upon which the mensuration of solids is largely based, "an oblique prism is equivalent to a right prism, etc." Such emphasis will help a student to get a proper view of the subject as a unified whole, an idea of the interdependence of the various parts, especially the relation of the minor theorems to the relatively few major ones. This will also help the student to retain his knowledge of the subject for later use.

I have tried to sketch rapidly and rather briefly some of the leading features in the movement to bring about better teaching in the subject of geometry. I might sum up what I have said in a general way somewhat as follows: So simplify the subject that

it may be of more educational value to, and more easily and fully comprehended by, the class of students to whom it is now generally taught; vitalize the study of geometry by again giving it a foothold in the soil of reality whence it sprang; but retain the systematic, logical treatment of the subject as an essential element in the training for the intellectual needs of life.

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A FEW GRAPHICAL METHODS.

By S. Douglas Killam.

Graphical methods in mathematics are not only of practical value, but are of great interest as they give the student a new viewpoint from which to consider, discuss, or solve some problem in pure mathematics. I will consider here an application of graphical multiplication to the solution of the quadratic equation $ax^2+bx+c=0$; the cubic equation $ax^3+bx^2+cx+d=0$; and in general the polynomial

$$ax^n+bx^{n-1}+\cdots+rx+s=0.*$$

Suppose we have two numbers a and b and we desire to find graphically the product $a\cdot b$. We mark off (Fig. 1) on a straight

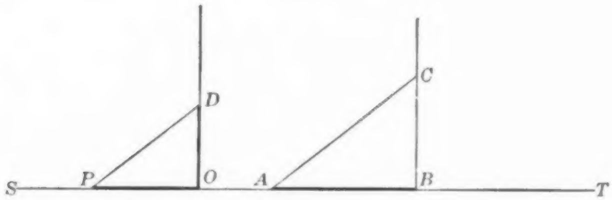

Fig. 1.

line ST, the distance $OP=$ unity, and $AB=b$. At O and B erect perpendiculars to ST and mark off $OD=a$. Through A draw AC parallel to PD, and we have BC equal to the product $a\cdot b$ measured on the same scale as b. We see this from the similar triangles DOP and CBA; where $CB/b=a/1$. If a is a negative number we measure OD downwards, and we have BC going downwards, and therefore represents $a\cdot b$ which is a negative number. If both a and b are negative we measure OD

* This method was first discovered by Captaine Lill, "Résolution graphique des équations numérique d'un degré quelconque à une inconnue," *Nouv. Ann. de math.*, 1867–1868.

downwards and AB to the left of A. This gives us a positive product represented by BC measured upwards.

To divide graphically we need only to reverse our process. We mark off $AB = b$ (Fig. 1) and $BC = a$; then through P we draw PD parallel to AC; and OD represents graphically the number a/b.

This method of graphical multiplication and division gives us a very simple method of representing the laws of multiplication and division.

Now let us consider the function $f(x) = ax^2 + bx + c$; and try to find graphically the value of $f(p)$ where p is any positive or negative real number. As before we mark off $OP =$ unity (Fig. 2) and $AB = a$. We then complete our construction as

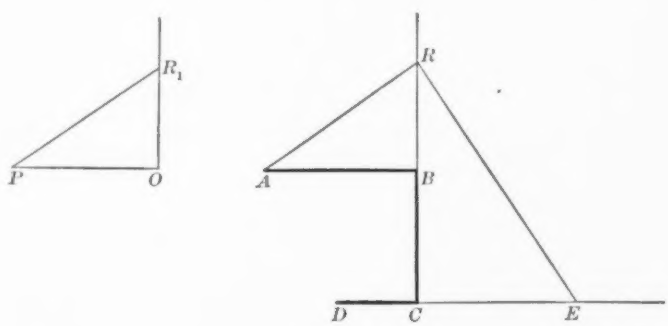

FIG. 2.

in Fig. 1, and see that BR represents in length $a \cdot p$. Produce RB to C making $BC = b$, then $RC = ap + b$. Through C draw CE perpendicular to BC, and through R draw RE perpendicular to AR; then from similar $\triangle$, EC represents $p(CR) = ap^2 + bp$. Produce EC to D, making $CD = C$; then $ED = ap^2 + bp + c$, which equals $f(p)$ our required result. If a is negative we measure AB to the left of A; if b is negative we measure BC upwards; and if c is negative we measure CD to the right. Now if p is a root of the equation $ax^2 + bx + c = 0$, then $ED = 0$, that is E and D coincide; so that in order to find the value p which makes $ap^2 + bp + c = 0$ we reverse our construction, starting at D or E, which coincide, and going backwards. Notic-

ing that the angle ARD was constructed equal to 90° we have a very simple way of finding the point R. On AD as diameter (Fig. 3) construct a circle, and where this circle cuts BC pro-

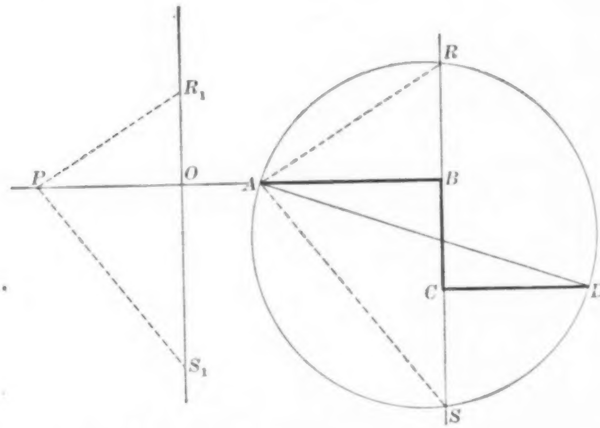

FIG. 3. Graphical Solution of $5x^2+3x-4=0$. Case 1, 2 real roots (unequal).

duced is one of our required points R. Now through P we draw PR_1 parallel to AR, and R_1O represents graphically one of the roots of the quadratic equation $ax^2+bx+c=0$. Our circle cuts BC produced in another point S_1 and if we draw PS_1 paralell to AS, we have OS_1 representing the second root of $ax^2+bx+c=0$.

Proof. Let p_1 and p_2 be the roots represented by R_1O and OS_1 respectively. Then

$RB=ap_1,$	$SB=ap_2,$
$RC=ap_1+b,$	$SC=ap_2+b,$
$DC=ap_1^2+bp_1,$	$DC=ap_2^2+bp_2,$
$DC+c=ap_1^2+bp_1+c=0,$	$DC+c=ap_2^2+bp_2+c=0.$

p_1 is a positive root being measured upwards; and p_2 is a negative root being measured downwards.

Corresponding to the three cases of the roots of a quadratic equation, (1) unequal real roots, (2) equal real roots, and (3) imaginary roots, we have three graphical constructions which illustrate clearly the three different cases; and this graphical interpretation gives the student a new idea of imaginary roots.

Case (1) (Fig. 3). The circle on AD as diameter cuts BC produced in *two* distinct points which give us our two unequal real roots.

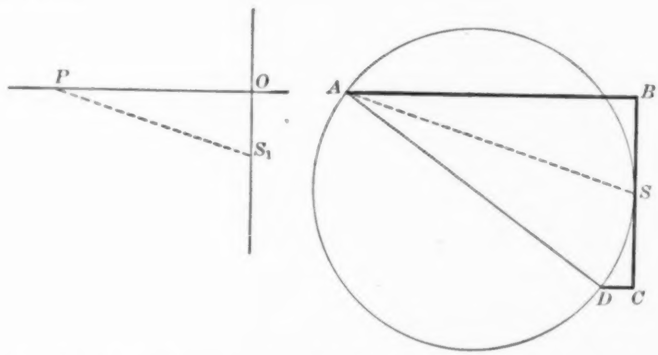

FIG. 4. Graphical Solution of $9x^2+6x+1=0$. Case 2, equal roots.

Case (2) (Fig. 4). The circle on AD as diameter just touches BC in one point (or two coinciding points) which gives us our two equal roots.

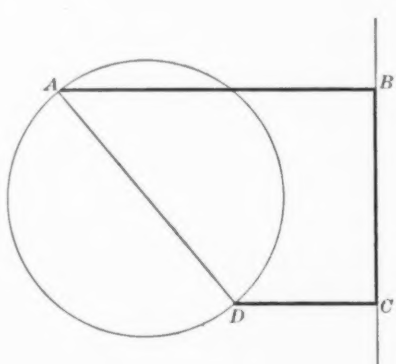

FIG. 5. Graphical Solution of $9x^2+6x+4=0$. Case 3, imaginary roots.

Case (3) (Fig. 5). The circle on AD as diameter does not touch or cut BC, and this corresponds to the case where the roots are imaginary. The accuracy of this method depends on the accuracy of our drawing and on the size of our unit of measurement. With a little practice and a unit large enough to make our drawing distinct, the results obtained will be accurate enough for most problems which involve the finding of the roots of a quadratic equation.

The roots (if real) of a cubic equation can be found in the same way. Fig. 6 illustrates the method of finding the value of

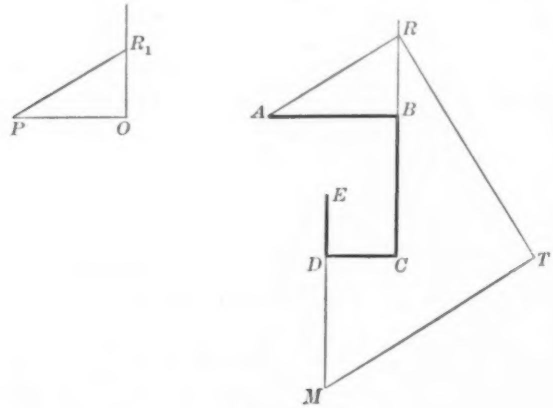

Fig. 6.

$ap^3 + bp^2 + cp + d$. If p is a root of our cubic equation M and E coincide, and we simply need to reverse our process.

$$BR = ap.$$
$$RC = ap + b.$$
$$TC = ap^2 + bp.$$
$$TD = ap^2 + bp + c.$$
$$MD = ap^3 + bp^2 + cp.$$
$$ME = ap^3 + bp^2 + cp + d.$$

Starting with the point E we have no simple method as with the quadratic equation of finding the roots. We must take some value p which we think approximates a root, and complete our

construction, and see if M and E coincide. If not take a better approximation and so on until M and E coincide.

In general the graphical solution of the equation

$$ax^n + bx^{n-1} + \cdots + rx + s = 0$$

is the same as with the cubic equation, but the greater the value of n the more complicated our work becomes.

Two linear equations with two unknowns.—

$$a_1x + b_1y + c_1 = 0 \qquad (1)$$
$$a_2x + b_2y + c_2 = 0 \qquad (2)$$

A very interesting graphical solution of the above problem was first given in an article by "van den Berg." His method is as follows.

On a line OD perpendicular to a line MN (Fig. 7) we mark

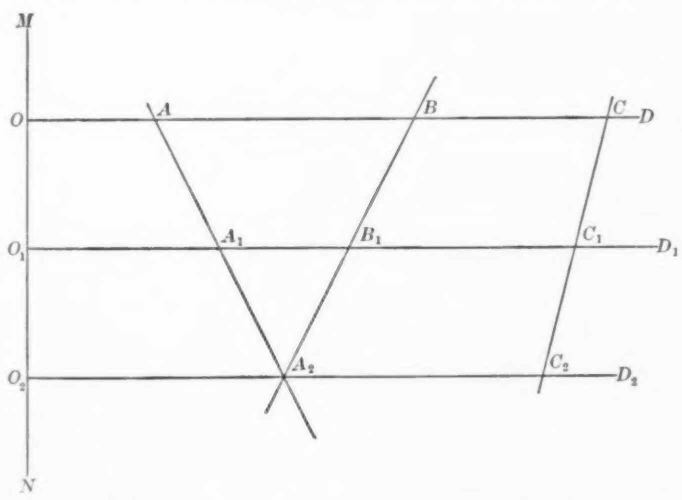

FIG. 7. Graphical Solution of $2x + 4y + 3 = 0$, $3x + 2y + 3.5 = 0$.

off $OA = a_1$; $AB = b_1$; and $BC = c_1$; on O_1D_1 also perpendicular to MN we mark off $O_1A_1 = a_2$; $A_1B_1 = b_2$; and $B_1C_1 = c_2$. Join AA_1; BB_1; and CC_1. Through A_2 the point where AA_1 and BB_1 meet draw O_2D_2 perpendicular to MN; then A_2C_2/O_2A_2

represents graphically the value of x which satisfies the equations above.

Proof.—Multiply (2) by λ and add to (1). Then $(a_1 + \lambda a_2)x + (b_1 + \lambda b_2)y + (c_1 + \lambda c_2) = 0$. We choose λ so that $b_1 + \lambda b_2 = 0$, that is $\lambda + -b_1/b_2$. From similar ▲ we see that $x = A_2C_2/O_2A_2$.

We could have eliminated the term in x by drawing a line through the point where A_1A_2 and MN meet. If AA_1 and BB_1 are parallel we must do this. If MN; A_1A; and B_1B are all parallel no solution exists. If MN; A_1A; B_1B and C_1C are all parallel then an infinite number of solutions exist. These graphically different cases can easily be interpreted in algebraic form. For three linear equations with three unknowns, we first eliminate one variable, and have left our problem with two variables. In the same way we solve n linear equations with n unknowns.

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ORIGINALS IN GEOMETRY.

By Harry B. Marsh.

In dealing with originals in geometry, or in any other branch of knowledge or activity, we can safely assume that it is human nature to like originals, to desire to make use of whatever information may be at hand in order to branch out along new and independent lines. This phase of human nature has played no little part in starting the great explorers of all times upon their epoch making expeditions; and who can deny that the discovery of this continent was due to the supreme effort of Christopher Columbus to demonstrate a world-embracing original in spherical geometry. This eagerness for individual achievement and success has been the inspiration of leaders and inventors, and it has been a very effective force back of all progress. It is present to a greater or less extent in all human beings.

By the time boys and girls have reached high school age, their liking to do original things is well developed and very evident. We see it displayed, for instance, in the manual training and domestic science classes, where the pupils listen to the directions of the instructor concerning the use of the various tools and implements employed, but all the while are impatient to begin doing and making things themselves. Is it logical to believe that this liking for originals is limited to the field of manual arts and sciences? By several tests with groups of boys the writer has satisfied himself that the natural liking for original work can be applied as well to mental as to manual work; to geometry as to carpentry. What is more important, he has found that he gets much better results by assuming that pupils ought to and do like originals than he ever could by thinking that they ought not to and do not.

The teaching of originals begins with the study of formal geometry, for the theorems and corollaries and definitions are the

(*Note.*—When this talk was given, all the points and methods brought out were illustrated by many figures which had been placed upon the board. Hence the following can serve but as an outline.)

working tools of geometry. The carpenter does not tell the apprentice simply that the chisel is a very sharp instrument of high grade steel. He explains that it is a handy tool to have when this or that piece of work is to be done. So, as we take up the different propositions, we should explain their uses and applications to certain kinds of originals. This can be done without breaking into the continuity of the subject, by introducing originals after small groups of theorems. After such a proposition as, "Upon a given line as a chord, to describe a segment of a circle in which a given angle may be inscribed," it is well to let the class know that this is often a handy construction to use when a triangle is to be built with the base and the angle opposite the base as parts of the given. It also has many applications in locus problems which have to do with a fixed angle. Then an exercise such as (C. E. B., 1908): "Let A and B be two fixed points on the circumference of a circle and P and Q the ends of a variable diameter of the same circle. Find the locus of the point of intersection of the straight lines PA and QB," furnishes an excellent application of this proposition and the ones that have preceded it in the measurement of angles. Many other theorems or groups can be treated in the same way, and made a valuable preparation for original work. Thus the pupils are made to feel that the relationship between theorems and originals is a close one.

When we consider originals themselves, we should keep in mind that our chief problem is not only to increase and encourage the natural liking of our pupils for them but still more important, not to diminish and discourage it. When once a boy looks upon all originals as hopeless, he is very likely to consider them as entirely beyond him and to make very slight attempt at solving them. The first and foremost cause of such discouragement comes from poor grading. If a class is led through a series of exercises which grow more difficult as fast as, but no faster than, their ability increases, they will attack problems that require no little mental effort with interest and confidence.

Originals should be looked upon as a pleasant break in the routine of formal theorems. They should be welcomed by both teacher and pupils alike. The attitude of the class towards them is generally a reflection of the attitude of the teacher. In order to be taken up with interest and enthusiasm by the pupils they

must be presented with equal or greater enthusiasm by the teacher. Even though, in the early months of the subject, most of the originals are taken from the text-book, it is possible to avoid a cut-and-dried assignment of them. In the latter part of first year geometry, however, and especially in review geometry, a great opportunity is offered in the line of practical problems and college entrance originals. "Here's a good one from Harvard," or, "See what you can do with this Yale question," produces at once both interest and inspiration. Such originals seem much more alive than those that have been lying dormant in some text-book for several years. In college preparatory review geometry divisions at least eighty per cent. of the originals assigned should be from recent entrance examination questions.

In teaching originals we shall do well to remember that we are trying to give the pupils a bird's-eye-view of the application of geometry. We should teach general methods; not simply special exercises. When we explain a difficult original to a class, it is more important to tell them how we know how to do it, than just how to do it, for we are dealing not only with that exercise, which the class may never have again, but with methods for similar originals which they are sure to have again. When an original is given to us our minds work along definite lines. Certain conditions in a hypothesis marshall up definite geometrical facts. The pupils should be given the advantage of this experience. They should be trained in right thinking. It is well to spend frequently a part of a period or a whole period in such drill work. Such questions as: How do you prove the products of two lines equal to the products of two others? By what means do you get a proportion? What are several ways of getting lines equal? lines parallel? angles equal? What kinds of loci do we have in plane geometry? What can you always do with an altitude in a construction? and many other similar ones, group together certain related facts in a useful way. Parallel lines not only suggest equal arcs to us, but also the substitution of one of the arcs for the other in the measurement of an angle. In this way we can frequently get two angles equal easily. In locus problems we generally try two or three special positions of the lines or points in question and get a very good indication of the solution; and so on. These are but illustrations

of many methods we have of attacking originals. The class should have the advantage of our experience and be taught and drilled on all these methods.

To emphasize and illustrate methods of solving originals, it is of advantage to take up many of them in class. In doing this our own figures should be accurate and general, if we wish to get the same kind from the class. If the conditions given should make angle A twice angle D, the figure should also do this. Otherwise, imagination as well as logic enters into the proof. The average boy or girl finds it hard enough to solve a difficult original without being handicapped. A poor figure is a big and a needless handicap.

When the exercise has been read or written on the board, all the hypothesis should be carefully studied, for a great many times failure to get an exercise is due to failure to use all of the given, every fact of which enters somewhere into the proof. Then if the solution of the exercise does not suggest itself right away have the class focus the attention upon the "To Prove," or "To Construct," rounding up all the possible methods of solution and then narrowing them down to the most probable. Check the tendency to introduce construction lines unless they are absolutely necessary, for many times they hinder rather than help a proof. Now ask for suggestions. There should be such a feeling of confidence and co-operation in the classroom that none should hesitate to propose his method. No suggestion should be rejected or accepted by the teacher without showing why it was or was not of use. By working together in this way occasionally, the pupils, if only from their habits of imitation, get hold of methods that are worth while.

The writer has also found that he gets better results if he supplements the home work on originals by frequent written class exercises. In these the pupil is not only thrown absolutely upon his own resources, but he is also compelled to so concentrate his mind that he can get the solution in a certain specified time. Numerous class exercises of this kind furnish excellent drill work for any who are planning to take entrance examinatione and it would be hard to find a satisfactory substitute for them. A division that has been made to write out many originals in class will be found to be much better trained and better equipped than one that has not had this practice.

In closing let me urge the importance of holding up the whole recitation for the boy or girl who has a different, and oftimes what proves to be a better proof than the one that has been given for an exercise. That boy may have worked hard and long the night before, and when he finally mastered the original he had earned not only the joy that comes from the successful completion of such a task but also the right to show his proof the next day in class. Everything else is worth giving up for the five or ten minutes the boy will use, for we are not only justly rewarding him for his efforts but we are furnishing an incentive and inspiration to the rest of the class to try to do as well.

TECHNICAL HIGH SCHOOL,
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THE PURPOSE AND THE CONTENT OF A COURSE OF MATHEMATICS FOR TECHNICAL AND MANUAL TRAINING HIGH SCHOOLS.

By Frederick W. Gentleman.

Introduction.

In the treatment of this subject I shall confine myself to those schools only that require a grammar school diploma, or its equivalent, for admission, that have a four year's course, and that are manual training or technical high schools as distinct from the "trade" school. I have confined my investigation, mainly, to public high schools, for with the institutions supported by the public the problem is more vital, and demands the attention of the public school teacher to a far greater extent than that of private institutions.

These limitations make the field which I am to cover differ much from the field covered in the Report on Mathematics in Technical Secondary Schools as compiled for the International Commission on the Teaching of Mathematics, for, not only will my discussion be along narrower lines, but I intend also to propose a course of mathematics, which seems to me to meet the demands of the times for such schools.

To this end, in order to get information concerning present actual conditions in manual training or technical high schools, or in high schools having manual training departments, I sent out a list of questions to a hundred of the largest public high schools in the United States, coming under one or another of the above heads.

From about seventy of these schools, representing twenty-eight states, I received more or less complete answers, of which twenty-five returned me answers quite complete, including such representative schools as: the Stuyvesant High School of New York City, the Rindge Technical High School of Cambridge, the Richard T. Crane Technical High School of Chicago, the Hughes High School of Cincinnati, the Denver Manual Training

High School, the Westport High School of Kansas City, Mo., the Minneapolis South High School, the New Haven High School, the North East High School and Southern Manual Training High School of Philadelphia, the Boardman High School of Seattle, the Springfield, Mass., Technical High School, and the Mechanic Arts High School of Boston.

The list of questions follows:

1. What was the total enrolment to your school last year?
2. How many were taking manual training?
3. If your course is a four years' course, of the class entering four years ago, the class of 1912, how many elected manual training?

The following questions are to be answered with reference to those who elected manual training?

4. At the beginning of the second year, how many returned as members of the class of 1912?
5. At the beginning of the third year, how many returned as members of the class of 1912?
6. How many were graduated last June?
7. How many of the graduates have entered, or expect to enter, some higher institution?
8. How many years of mathematics are required of the manual training pupils?
9. What branch of mathematics is taught the first year? *a*. Periods per week? *b*. Text?
10. What branch of mathematics is taught the second year? *a*. Periods per week? *b*. Text?
11. Comment upon other courses in mathematics offered to manual training pupils.
12. Is any effort being made to correlate the different branches of mathematics?
13. If so, to what extent?
14. Is it the policy of the school to encourage, or to discourage, preparation in the manual training course for higher institutions?
15. Is your school coeducational?
16. If so, is the same mathematics course required of both boys and girls?
17. Remarks.

The purpose of the first two questions was to learn the size of the school and the proportion of the pupils taking manual training, in order to judge the value of the answers that follow, hence, no more comment is needed concerning them.

Furthermore, the results of the last two questions, 15 and 16,

I will discard as useless, as I found that, in all schools that are coeducational, practically the same mathematics is offered for the girls as for the boys, excepting that in most cases less is required of the girls.

The results of questions 7 through 14 I will discuss as needed, but questions 3 to 6, inclusive, which deal with the falling off in attendance from year to year, show a condition that requires most serious attention.

By averaging the statistics of the twenty-five schools from which I received answers to these four questions, I find that, of every hundred pupils who entered the lowest class in 1908, but sixty-three returned the second year, but forty returned the third year, and but twenty-seven were graduated. The decrease is therefore far greater in the first two years than in the last two years of the course.

Hence, noting that out of every hundred pupils sixty leave during the first two years, my problem is to propose a course of mathematics: (*A*) that will give as much power as possible to those pupils who can remain two years or less, which I shall call *short time pupils;* (*B*) that will furnish for those who *can* remain throughout the course an incentive to complete the four years—*possible graduates;* (*C*) that will render it possible for those desiring to enter some higher institution, to be able to do so—*college preparatory pupils.*

A. Short Time Pupils.

Of these there are two classes: (*a*) Those who lack the ability to perform the work of high school grade, most of them older than the average and delinquent; (*b*) those who, because of economic conditions, must leave school to go to work.

For group (*a*): inasmuch as it is not the function of the school to supply ability, and as this group, up to the present, has not shown evidences of ability to acquire more than has already been acquired in the grammar schools, I shall not consider its special needs in my discussion, believing that the few who have not already been directed to trade schools should be allowed to get what little they can from the course designed for the more capable ones.

For group (*b*): first, what are the ends we should aim to at-

tain? To me the following seem most important: power to think clearly, habit of doing work accurately, rapidly and neatly, and spirit of self-reliance; also, for those who can stay the major part of the two years, ready skill in the control of necessary mathematical ideas to forward the technical problems in hand, habit of functional thinking, acquaintance with computing methods and significant figures, and methods of minimizing labor in calculations.

The means for securing the above ends I shall consider under the following heads: I. Arithmetic, II. Algebra, III. Geometry, IV. Trigonometry and Tables, V. Applied Problems, VI. Correlation.

I. *Arithmetic.*—It is true, that, in general, the *short time pupils* will have actual use for no mathmetics beyond the simpler calculations in arithmetic. Hence, it is my conviction that arithmetic should occupy an important place in the course for these pupils. First of all, it should serve to clinch the work in arithmetic already given in the grammar schools; then, through substitution in formulas, it should lead the way to algebraic forms; through graphical tabulations, lead the way to negative numbers; and, through mensuration, lead the way to a fuller acquaintance with the properties of geometrical figures. At no time in the course should there be an extended period, when practice in arithmetical computations should be lacking.

II. *Algebra.*—First, let us consider the value of training in algebra. Prof. Hedrick, of the University of Missouri, in an address before the National Educational Association in 1910, said: "The symbolism of algebra has direct value, for we can express in brief forms the facts of science and everyday life, and work with these forms by abbreviated processes. Algebra is made up of operations upon relations between quantities—the study of variable quantities, relations between these variable quantities, and acquisition of ability to control and interpret such relations. Now the very essence of science is to discover the precise dependence of changes in one quantity upon changes in another, which Prof. Klein calls 'habit of functional thinking.' The spirit of algebra is the dependence of varying quantities upon each other—cause and effect."

Furthermore, in the Report of the Committee on Mathematics

in the Public and Private Secondary Schools of the United States is found the statement that the general idea underlying the teaching of mathematics is the development of the scientific aspect.

Hence, if we are to give our *short time pupils* any training in pure mathematics—and, under the head of Applied Problems, I shall show that we should—I claim that this training should be, in the main, with algebraic processes.

The results of my investigation will show to what extent this training is being given. Of the fifty-eight schools answering question 9 concerning the mathematics for the first year, fifty-two offer algebra, three algebra and arithmetic, and three offer algebra and geometry, using as a text Short and Elson's "First Year Mathematics." The texts are distributed as follows:

Hawkes, Luby and Touton	12
Wells' Essentials or Academic	15
Wentworth	9
Milne	10
Young and Jackson	4
Slaught and Lennes	4
Marsh	4
Hull	1
Hart	1
Stone and Millis	1

Now, as I consider this array of texts I am convinced that over-emphasis is placed upon theory. Much of the work now found in these texts should be omitted, not only for the first year, but for the first two years, and it should be replaced by problems connected more closely with allied subjects. Modern industrialism, with its demand for tangible success, has led to a great outcry for more practical school work, and has registered its contempt for "mere theory." We must give work to the pupil that is as practical as possible, also that is at the same time interesting to him. Prof. Bennett, of Columbia, says that even the cultural value of a subject depends upon the extent to which that subject can be, and actually is, linked with the activities and thought content of real life; that in this way the old static, historic, idealistic conception of mental discipline is being replaced by a dynamic, realistic, practical view.

The value and place of the practical problem will be discussed at greater length under the head of Applied Problems.

III. *Geometry.*—The fact that the catalogs of all "trade" and "industrial" schools, to which I have had access, require a knowledge of many of the properties of geometrical figures, alone gives that subject a place in the course. The pupil should have drill in the mensuration of plane and solid figures, should know how to construct many of the plane figures in common use, and through applications should become acquainted with many other properties of geometrical figures besides areas and volumes, such as those of congruent and similar figures. In an informal way he should be led to see the connections between many of the statements of geometry, in order to aid his memory of them and to give him some feeling of security as to the truth of the facts.

Now, as to the actual conditions at the present time, I received answers to question 10 from fifty-eight schools. Of these, forty-eight offer plane geometry alone; six offer algebra and geometry—of these six, three use Short and Elson's "Second Year Mathematics," and the other three offer parallel courses; two schools continue algebra and arithmetic, offering plane geometry the third year—the Detroit Central High School, and the Beloit, Wisconsin, High School; and two offer plane geometry and shop mathematics—the Newark Technical School, and the New Haven High School.

The texts for geometry are distributed as follows:

Wentworth and Smith	10
Wells	7
Shutts	6
Stone and Millis	3
Milne	3
Phillips and Fisher	2
Durell	2
Saunders	2
Shultze and Sevenoak	2
Slaught and Lennes	1
Bush and Clark	1
Robbins	1
Lyman	1

Noting these courses offered, it is evident that great emphasis

is laid upon formal geometry. I consider this a very weak point, for formal geometry, whatever may be its value in developing the "logical faculty"—and this is denied by some educators—is a very difficult subject even for the best pupils, and is to many nothing else than an exercise for the memory. In this connection I quote from Prof. Young's address before the Association of Mathematical Teachers in New England: "Formal methods of reasoning with continued explicit reference back to a hypothesis is not the natural method of reasoning to high school pupils. . . . A really formal proof in a high school course is utterly impossible." These views, expressed with reference to formal geometry for all high school pupils including even the pupils preparing for college, apply even more strongly to pupils in technical schools. Hence, I must conclude that a course of such doubtful value as formal geometry has no place whatever in the course of mathematics for the *short time pupils*—these pupils, many of whom must because of economic conditions go out so soon to begin the battle of life.

IV. *Trigonometry and Tables.*—For practice in accuracy and rapidity of arithmetical computations trigonometry offers a good field. Moreover, it is useful in impressing upon the pupil the idea of ratio, and the equality of ratios of corresponding sides in similar figures. Experience in class rooms with low grade divisions has shown me that the pupils comprehend trigonometry much more quickly than they do formal geometry. Again, it offers a field for many interesting and useful problems.

The use of tables in the course is for the direct purpose of giving the pupil some acquaintance with methods of computation, by which he may minimize his labor. He should be taught to use the square root and cube root tables, logarithmic tables to four places and trigonometric tables to four places, and he should be taught to what extent results obtained by the use of these tables are of practical value.

V. *Applied Problems.*—Under the head of Algebra I have already brought out the over-emphasis on the theoretical, and the demand of the industrial world for the practical problem. Recognizing this demand for problems from real life, as one must, I will now consider the kind of problem needed. I doubt very much if problems about areas of states, about populations,

planet and star distances, imports and exports, latitudes and longitudes, linoleums and ornamental designs, etc., are any more real, or of much more practical value, to the boy than the much condemned "cistern," "work," and "hare and hound" problems. Few of such problems have a place, but the real applied problem should be, I believe, a problem that might reasonably occur in the pupil's own actual life. Many of this sort should be brought into the early part of the high school course to prepare the pupil, soon to leave, to do real work of the kind similar to the problems learned. Continued use of mathematics in real problems gives the pupil, first of all, an idea that mathematics can be used, and, also, develops some judgment as to how to use it. He will come to regard mathematics as a valuable tool that can do efficient work.

Because of the enthusiasm for, and the evident demand for, real problems to the exclusion of theory, there has arisen in England the Perry Movement, and with it the neglect of pure mathematics. At the convention of the National Educational Association in 1910, Mr. William Breckenridge, of the Stuyvesant High School, New York City, speaking on "The Perry Movement," said: "We must keep in mind that pure mathematics is, after all, the thing the student much needs. Real problems are unorganized and ungraded, so that the study of them leads nowhere. The student is hurried through a mass of problems and finds on their completion that the product of the work is a confused notion of everything in general and no clear idea of anything." Further than that, that many schools, which have complete courses for training for the industries, see the need of pure mathematics is evident from quotations from their catalogs. In that of the Newark Technical School is the following: "From an educational standpoint the study of mathematics is essential because of its disciplinary value in forming habits of attention, of concentration, of accuracy and precision of thought. . . . It is equally necessary from the economic and practical standpoint. Without a knowledge of mathematics but little, if any, progress can be made in any branch of physical science. . . . Men with a knowledge of mathematics are needed in the machine shop, in the draughting room, and in the field."

In the catalog of the David Rankin, Jr., School of Mechan-

ical Trades, St. Louis, Mo., is found the requirement that all day students shall take courses in arithmetic, elementary geometry, formulas and solution of problems involving one or two unknowns, mechanics, and elements of trigonometry.

Some reformers are urging that the mathematics of the technical schools should follow the lines of the Perry idea, a condition which already exists in many of the industrial and trade schools. To show to what extent this is being done in certain industrial schools, and at the same time to show of how little value such methods of training are to the student, I quote the following from the Supplementary Report on the Industrial Schools of Secondary and Intermediate Grade for the International Commission: "In certain industrial schools mathematics, though definitely included in the course, is not taught as a separate and distinct subject, but introduced as the student strikes some phase of the work in the shops or draughting rooms requiring knowledge of a certain fact, which is brought out for immediate use. In this way he obtains his mathematics. . . . In the first place, such instruction can hardly develop originality on the part of the pupil, and, in the second place, he has not the apperceptive mass from which various mathematical facts and relations can be drawn out. At best he can only be made to see that the statements made to him are plausible. At most he sees only a glimmer of light, and then comes total darkness. When one realizes how difficult to most pupils are certain propositions in geometry, what must be one's judgment upon a method of teaching which tries to impress isolated mathematical facts upon the mind—upon a mind, moreover, which has not been prepared by constant drill to recognize intuitively mathematical relations? The conclusion must be that such teaching can not give the pupil power or lasting knowledge."

If such is the conviction of the committee concerning the purely industrial schools investigated by them, certainly pure mathematics has a place in the technical high school. Although present conditions in these schools show that too great emphasis is placed upon pure mathematics, and although, on the other hand, there are those who would eliminate pure mathematics as such from the course entirely, I contend that a reasonable middle course can be and should be made.

VI. *Correlation.*—To gain ready skill in the control of necessary mathematical ideas, it appears to me impossible to treat the subjects named as separate sciences, but such parts of arithmetic, algebra, geometry and trigonometry must be selected as are fundamental for such an end. Mathematics should not be a series of discrete subjects, each in turn to be studied and dropped without reference to the others or to mathematical problems that arise.

In my investigation I found that, of fifty schools answering questions 12 and 13, eighteen made no effort whatever at correlation, twenty made but slight effort, and twelve made much more effort in that direction, but practically none correlated the work during the first two years, excepting the three using Short and Elson's "First and Second Year Mathematics," and the Hughes High School of Cincinnati.

As I consider the ends for which I believe we should aim for the ***short time pupils,*** I do not accept as a valuable means the fusion of algebra and formal geometry. I have already shown why I do not believe that formal geometry has a place in the course for these pupils. Furthermore, concerning the fusion of algebra and formal geometry, Prof. J. W. A. Young, of Chicago, speaks to the point when he says: "It seems to me that the fields of algebra and (formal) geometry are essentially different, both in ground covered and in methods used. These differences seem sufficient to preclude the possibility of the fusion of the two into a homogeneous whole that shall be neither algebra nor geometry, but a real composite of the two."

That many leading educators in this country are seriously considering the need of correlation of the different branches of mathematics in the secondary schools, there is but little doubt. We need to break down the watertight compartment idea of mathematics being made up of a number of isolated and unrelated subjects. We need to unify more fully all our mathematics."

My discussion, up to this point, has shown that, in general, the mathematical courses in technical and manual training high schools in the United States differ in no important educational feature from those in the classical high schools, that the courses are designed, not primarily for the majority of the pupils enter-

ing the school, but for the group of pupils using the high school as a step towards college preparation. I have shown that to meet the needs of this larger group, the *short time pupils*, certain modifications of the mathematical courses are necessary. For this purpose I have considered the values of arithmetic, of algebra, of geometry, and of trigonometry. I have shown the need of problems from real life, bringing out the danger of over-emphasis in this direction. I have shown the need of training in pure mathematics. I have shown that formal geometry has no place in the course. Finally, I have shown that, in order to get the greatest value out of the course for the first two years, the different branches should be combined into a single course of mathematics.

B. Possible Graduates; Those Who Can Stay if They Will.

The main purpose for the first two years in reference to this group is to have the course of such nature as to furnish a strong incentive to remain in the school. It is obvious that the work should be simplified and adapted to the ability of the immature pupil of the first year, so that he will not become discouraged at the beginning of his high school course. It is also obvious that the subject matter should arouse the interest of the pupil, and that he should feel that it is of such a nature as to be of value in meeting his needs. In an address by Prof. W. S. Munro, of the University of Missouri, I find the following: "Unless a pupil feels that there is some reason for studying a subject, unless a pupil has a motive for studying a subject, our efforts to teach that subject are practically fruitless. The only satisfactory motive for the study of mathematics in the first year or two of the high school must be based upon, or connected with, a feeling that the subject matter of mathematics is worth while or valuable in itself, that it is useful, good for something." Hence, the pupil's course should be vitalized by many problems taken from actual conditions about him. He should not be called upon to master a collection of abstract truths, to master the fine distinctions of a logical demonstration, which is so ill-suited to his powers, when the reason for so doing is that he

may gain only a promised but shadowy mental discipline. The belief that mathematics is of real worth should come to the pupil early, for which reason he should see as much of the field of mathematics with its applications as possible. The subject matter should be so arranged and problems of such sort that he may see in his different courses some unity, see the connection of the different branches of mathematics with each other, with his drawing, with his science, and with his shop practice.

As a means of saving time, of decreasing labor in calculations, use of square root, cube root, and logarithmic tables will commend itself to the pupil.

If the course of mathematics for the *possible graduates* for the first two years is based upon the above ideas, a strong incentive will thus be offered them to remain.

Before considering the content of the course for these pupils for the last two years, we must have before us the purpose of the technical high school. From the Report of the Committee on Industrial and Technical Education in Secondary Schools, presented at the meeting of the National Educational Association in 1910, we find the following: "A technical high school has for its distinct purpose the preparation for industrial leadership, positions requiring skill and technical knowledge. The instruction deals not only in important manual operations, but also with principles of science and mathematics and their direct application to industrial work." In another part of the report we find: "Its (the secondary technical school) main object is the preparation of pupils for efficiency in a large group of important positions in industrial life, aiming to cultivate industrial intelligence, these qualities essential for efficient industrial leadership rather than abstract reasoning power." Hence arises the need that a pupil learn how to attack a specific problem, how to analyse it, and how to select the necessary tools for solving it.

Now, let us note what the schools investigated are doing in order to gain such results. Of the fifty schools answering the question 11 concerning the mathematical courses offered beyond the second year, thirty-six named college preparatory subjects only, that is, solid geometry, trigonometry, and advanced algebra—of these only nine claimed to attempt correlation to any degree; two schools offered commercial arithmetic; three

offered analytical geometry, three surveying, and six a definite course in shop problems.

From these statistics it is evident that by far the majority of these schools are offering not much else than the course laid out for them by the colleges. Here again, I am convinced that undue emphasis is placed upon pure mathematics, to the exclusion of applied work. I would continue work in pure mathematics for the valid reasons given before, but I would replace a large share of it by a definite course in applied mathematics. This course should include problems from physics, from chemistry, from the drawing department, and from the shops.

As to the nature of the shop problem, if it is not taken directly from the shops, it should, at least, fit the conditions existing in the shops; it should be stated in terms of the shop as far as practicable; it should furnish results approximated to a required degree of accuracy that represent good shop usage; and it should not be made unnecessarily complicated so as to make solution more difficult.

Part of this course of applied mathematics should be devoted to acquainting the pupil with the use of surveying instruments. Much practice should be required in the use of different kinds of tables and of other time saving devices.

In discussing the course for the *short time pupils*, my aim was to develop the power of thinking clearly, habits of accuracy, rapidity and neatness, habit of functional thinking, ready skill in the control of necessary ideas for meeting the problems of life requiring the use of mathematics. The course for the *possible graduates* should be of such nature as to continue to develop these same ends.

C. College Preparatory Pupils.

For these pupils the problem is not what mathematics shall be offered, for that is determined by the higher institutions, but, first, why pupils should be allowed to prepare for college in technical high schools, and, secondly, in view of the course that I would offer the first two years, how the course of the last two years could be arranged to meet college requirements.

I. From the Report of the Committee on Industrial Education before the National Educational Association in 1910, I

quote the following: "Many educators feel that no system of education should be allowed to develop blind alleys, and wish to see the way kept clear for any youth to pass from one school to the next higher." Again, from the Report of the Committee of Nine on the Articulation of the High School and College before the National Educational Association in 1911, I quote the following: "The high school period is a testing time, a time for trying out different powers, a time for forming life purposes. The opportunity should be provided for the student to test his capacity in a fairly large number of relatively diverse kinds of work. In the high school the boy or girl may very properly make a start along the line of his chosen vocation, but a final choice should not be forced upon him at the beginning of that career. If he makes a provisional choice early in the course, there should be ample opportunity for readjustment later in the high school."

From another part of the same report, in order to show the value of the technical school training for those who may continue school education, I find this: "The organic conception of education demands the early introduction of training for individual usefulness, thereby blending the liberal with the vocational, for only then does liberal education receive its social significance and importance. The boy who pursues both the liberal and the vocational sees the relation of his own work to the work of others and to the welfare of society."

Hence, it is seen that many leading educators believe that, after the high school course is begun, there should still exist the opportunity to go on to institutions beyond the high school, and that the training received in such schools as technical high schools is of distinct value to such pupils.

In addition to this belief, there is a demand for such opportunity, evidenced by the fact that so many of the graduates of technical and manual training high schools do enter higher institutions. From the thirty-five schools answering question 7 in my list of questions, I find that 35 per cent. of the graduates, more than a third, enter higher institutions. This is one per cent. higher than the average for all public high schools in the United States, as found in the Report of the Commissioner of Education, for the year 1910. I find, too, that, of the fifty-six

schools answering question 14, it was the policy of forty-two to encourage preparation for college in the manual training departments.

Many a pupil does not find himself for some time after he enters high school, for it is then that he is just entering upon the period of adolescence, and, as the high school subjects are, in the main, new to him, it is only by the actual study of them that he can know whether he will find them interesting to him and whether he can master them.

Before the Association of Mathematical Teachers in New England, at the December meeting in 1912, Prof. D. E. Smith said that we must keep an open door in mathematics, because we know not what mathematicians may be among our numbers, so I claim we must keep the open door in all high schools for higher institutions, for we know not what scholars may be among us.

II. Assuming that we do recognize the necessity of allowing pupils entering the technical high school to prepare for college, now comes the problem as to how to arrange the course to make it possible.

As I have planned the work in mathematics, and have held in mind the remaining subjects of the course for the first two years, the college preparatory pupils, in general, have not been considered. Still, the work in English, in elementary science, in history, with the probability of one year of a modern language, and in mathematics would serve as a very good foundation upon which to build.

In mathematics, the only problem to offer a real new difficulty would be the formal demonstration in geometry, for the pupil should have been well informed about most of the properties of geometrical figures, and should have had enough theoretical work in algebra to serve as an equivalent of the work usually done in algebra in the first year of the average high school.

At the beginning of the third year should come the time for the entire separation of these pupils from the *possible graduates*, for, by that time, most pupils will have come to know which course is most fitting for them.

Let us suppose that a school has a program providing for thirty periods a week. For the third year, I would divide it

as follows: Mathematics, 5; Modern Language, 5; English, 5; History, 3; Physics, 5; with an elective, if desired, in the other Modern Language, Drawing, or Shop Practice, 5 or 6. For the fourth year, I would make the following division: Mathematics, 5; Modern Language, 5; English, 5; History, 3; Chemistry, 5; with an elective, if desired, in the other Modern Language, Drawing, or Shop Practice, 5 or 6.

Based upon the requirements I have laid down in the preceding discussion I will now propose briefly what seems to me a reasonable content for the course of mathematics for the schools discussed:

A. First Year.

I. Plane figures. Mensuration of those familiarly known to the pupil, giving practice in use of fractions and mixed numbers. Board measure.

II. Construction of plane figures, using for instruments compass, ruler and protractor. Drawing to scale of plans for models and of polygons.

III. Square root. Formulas from geometry and the sciences.

IV. Addition and subtraction of algebraic expressions. Graphs of statistics. Problems from geometry.

V. Equations. Parentheses. Problems.

VI. Multiplication of algebraic expressions, special products, equations, problems.

VII. Division using monomial and binomial divisors only. Equations.

VIII. Simultaneous linear equations. Problems. Graphs.

IX. Indexed list of definitions and principles of geometry used, also of formulas.

B. Second Year.

I. Review of algebraic operations.

II. Factors:

- (*a*) Monomial factors.
- (*b*) Binomials—difference of two squares, sum or difference of two cubes.
- (*c*) Trinomials—squares and cross products.
- (*d*) Polynomials of four terms containing a common binomial factor.

III. Quadratics having rational roots, both of one and two variables, solved by factoring. Problems.

IV. Common factors, common multiples, fractions, equations.

V. Roots, use of square root and cube root tables. Formulas. Radicals containing monomial quadratic surds only.

VI. Quadratics having irrational roots, solved by completing the square or by formula. Problems.

VII. Ratio and proportion. Similar polygons. Trigonometric functions of acute angle of a right triangle. Similar solids.

VIII. Exponents and logarithms. Only the treatment of exponents necessary for intelligible use of logarithms.

IX. Trigonometry. Solution of right and oblique triangles.

X. Indexed list of formulas and of principles of geometry used. Square root, cube root, logarithmic, and trigonometric tables.

C. Third and Fourth Years.

I. *Possible Graduates.*—Since the course for those who finish their formal education in the high school should be as closely correlated as possible with the work of the shops, drawing room, and science laboratory, I shall make no attempt to fix the order in which the work should be taken up, as that would depend much upon the order in which the subjects are taken in the other departments.

I believe that the course should be divided into two rather distinct parts, one part consisting of pure mathematics mainly, and the other part a course in shop problems, alternating with the first, if possible. For the first part, trigonometry with surveying, followed by algebra, should be offered. For the second part, there should be problems on simple machines, problems on weights of bars of various shapes and materials, on strength of materials, problems on horse power of engines, motors, belting and shafting, problems on hoists, milling machines, hand and engine lathes, lathe indicators, safety valves, heating surfaces of boilers, etc. For the solution of these problems the pupils should be taught the use of the slide rule and of all tables that

will minimize labor. In other words, the work in mathematics should be both pure and practical, the practical being as closely connected with the work of the mechanical departments as possible.

II. *College Preparatory Pupils.*—During the third year formal plane geometry should be given and completed so that the pupil will be prepared to take preliminary examination at the end of that year.

During the fourth year the course should include solid geometry for about twelve weeks and algebra for the remaining time, covering the college requirements for advanced algebra.

At the present time one radical change in our educational system that is being advocated by many prominent educators is the division of the first twelve years of the school course into two periods of six years each. This change has the support of such a well-known educator as Prof. David Eugene Smith, of Columbia, and also was urged in the Report of the Committee on Economy of Time in Education before the National Educational Association in 1911.

Another radical change that is being proposed is the raising of the compulsory attendance age limit from fourteen years to sixteen or eighteen years. Dr. Franklin H. Dyer, Superintendent of Schools, Boston, Mass., is a strong advocate of this change, but along with it he is advocating part time schools, especially for pupils beyond fourteen years.

Now the course I have outlined would, I believe, fit very well into either or both of these schemes, because much emphasis is placed upon constant drill in arithmetical computation, because the course is continuous, and because the subject matter has been so selected as to make the mathematics worth while for the solution of problems likely to be met in everyday life.

The experiment with the part time school, begun by Superintendent Dyer, has been tried, at some length, in the Hughes High School of Cincinnati, with apparent success. Along with it has been developed a course of mathematics for these pupils that is in many respects similar to the one proposed above for all pupils in the technical high schools.

In conclusion, viewing the many problems at present needing

solution in the technical and manual training high schools, I contend that if the mathematical course for these schools follows along the lines advocated above, they will be accomplishing to far greater extent than at present what they should for the best interests of the pupils, hence, will be giving the coming generation a training that will make them far better and more efficient citizens.

Mechanic Arts' High School,
Boston, Mass.

EFFICIENCY VS. THE INDIVIDUAL.

By Leonard M. Passano.

I.

One of my colleagues, in a recent letter to *The Nation,* caustically deplores the prevailing practice of "The Czar," of college presidents, and "even of the deans"—I am quoting his words—their practice of discussing, in public and in print, the sins and shortcomings of the teacher. He says that, just as the inefficiency of the saleswoman in a department store is discussed by the management in its office and in private, so the faults and failures of the teacher should be talked about, *sub rosa,* in the presidential office or in the sanctum of the Carnegie Foundation.

I do not agree with my colleague. The inefficiency of the saleswoman concerns, primarily, her employer and herself, and only secondarily the customer and the public. The efficiency or inefficiency of the teacher concerns primarily the customer, the pupil, and the public. The teacher owes his duty first to the cause of education; secondly, to the people; thirdly, to the pupil; and fourthly, to his school. The whole duty of the educational management also, whether college president or school superintendent, consists of these same four items. Moreover, the management and the teacher owe mutual duty to each other as constituents of the educational democracy.

Such being the case the teacher, as also the management, is a subject of public interest, a subject for public discussion; and my colleague's error is simply another instance of the many fallacies arising when one attempts to apply, as he says, "the lesson of business administration" to "academic offices."

However that may be the teacher is now, always has been, and, probably, ever shall be publicly discussed. In the seventeenth century the author of "The Anatomy of Melancholy," quoting from Virgil and Horace, pictures the scholar and teacher in his prime as attended familiarly by

"Grief, labour, care, pale sickness, miseries,
Fear, filthy poverty, hunger that cries,
Terrible monsters to be seen with eyes."

And in his decay:

"At last thy snow-white age in suburb schools,
Shall toil in teaching boys their grammar rules."

If Democritus Junior, if Horace and Virgil have said that the teacher is undervalued and underpaid, it is not seemly for us to deny it. Nor is there any disposition in current discussions to deny these facts. They are universally acknowledged, generally bemoaned, and for the most part treated as unalterable laws of nature. Nor do I consider them the most important items in this discussion. Of far more importance are the accusations of inefficiency and futility brought against the teacher and his methods; of importance, not because of the element of truth which is in the criticism—for it is not more true of education than of any other human activity—but because of its reiteration by men who are ignorant of the real aim of education, or who wilfully misconstrue that aim for their own ends.

It seems to the present speaker that there are three paramount influences leading to this criticism:

1°. Over-organization of the educational system;

2°. The demand from the "practical" man for a "product" which the true educator refuses to "produce";

3°. The abnormal growth of "efficiency" management in the industrial world.

These three influences are not mutually exclusive, but it is convenient to treat them separately.

II.

Over-organization would seem to be inseparable from the modern social system. That question we need not discuss. But the narrower question of its inseparability from the present educational system need not be answered unreservedly in the affirmative. Over-organization would seem to be largely a result of overgrowth, and overgrowth would appear to be preventable. It is avoidable at least in the technical school and the

college. Whether it can be avoided in the public school system seems a more debatable thesis. Accepting such organization, however, as inevitable, it does not follow that the faults of over-organization cannot be corrected.

First among these faults is the belief that organization can replace the individual; that the teacher, trained according to some perfect system and made a unit in some perfect school organization, will go on performing his functions perfectly, like a piece of automatic machinery, indefinitely; or at least until he is worn out and can be scrap-heaped.

Of course the parts of the machine must be lubricated, and at times readjusted. So the engineer goes travelling from Dan to Beersheba and back to Dan again seeking new lubricants.—We may, perhaps, pause long enough to say that lubricants are not funds or endowments.—He goes journeying about, when he might better remember that the parts of his machine are men and women working upon a thing of infinite variety, the young or adolescent human animal; and that oftentimes he would be better employed at home in trying to know and to help the individuals of whose company he is one.

If he did so he might doubtless be called "inefficient," and his doings would be no longer widely chronicled in the journals which help to spread the disease of our age, mechanistic megalomania. Perhaps, however, he would be recompensed by the more permanent chronicles of fifty or a hundred years hence. But the organization demands that he go to the Atlantic coast to learn that we are squandering x dollars per student-hour on algebra or Latin, and to the Pacific to learn that the teacher's useful work per week equals N multiplied by H. Thus the traveller brings home these "many inventions," and finds others awaiting him there.

I should be the last to discourage a change to improvement, but with Bacon I believe, "It were good therefore, that men in their innovations, would follow the example of Time itself; which indeed innovateth greatly, but quietly, and by degrees, scarce to be perceived. . . . And well to beware that it be the reformation that draweth on the change and not the desire of change that pretendeth the reformation."

Now change is desirable if change be normal and healthy

growth, and such normal and healthy growth the "good" individual teacher attains. But with him it is *growth*, slow but natural. Too often the organization change is a piece of skillful surgery. A leg is cut off and in its place two arms are made to grow; or a leg and an arm are cut into bits and remolded into a limb. Some wonderful freaks are produced, but like other abnormalities they fail to propagate their kind. Their number certainly serves to measure the activity of the educational management, and might indeed be used to measure its efficiency by adopting some such definition as the following: efficiency of management equals the number of new ideas learned divided by the number of new ideas adopted.

This propensity to consider new ideas (or old ones revised) made into a system, and worked by an organization, as a self-sufficient automatism in human activity, is not confined to educational matters nor is it entirely novel. Two centuries and a half ago William Penn said what will, with the alteration of a word or two, fit our present case:

"Educational organizations,"—he said governments—"like clocks, go from the motion men give them; . . . Wherefore [organizations] rather depend upon men, than men upon [organizations]. Let [teachers] be good, and the [educational organization] cannot be bad . . . but if [teachers] be bad, let the [organization] be never so good, they will . . . warp and spoil it."

One does not deny that the teacher to-day is better trained and better fitted for his work than he was fifty years ago or twenty-five years ago. It would be strange if he and his methods did not improve. He is accused, indeed, of being reactionary, of resisting all innovation. On the contrary, he is eager for improvement, and he keeps pace with his age. But he knows the past as well as the present, and knows that upon both the future must depend.

The "efficient" teacher of the educational organization has no concern with present, past or future time in education, except to condemn the past, to monopolize the present, and to hypothecate the future. His are the eternal laws of psychology and of pedagogics; given which, he knows exactly what to do with a class of a dozen or two-score "mental states" or "objects of

consciousness"—or by whatever name he calls the college freshman or the grammar school child. The "good" teacher has his psychology, too; but he calls it, or mixes it with, common sense. He faces a new class of a score of human beings knowing that in all probability it will be much like the last, but knowing also that it will have its differences; and he trusts to his own knowledge of human nature for adjustment to this new group of personalities, to his own individuality for the comprehension of theirs.

There is no reason why a teacher should not be both "good" and "efficient," for not even over-organization can completely obliterate individuality; but there are far too many who, being technically efficient, are individually unfit. They are an efficient part of the school mechanism, but to the child and to the child's parents, who after all are the people, they are a thing, a "brain . . . as dry as the remainder biscuit after a voyage."

The "man on the street," recognizing that something is at fault but being taught to believe in organization—being himself, most probably, a unit of one—the man on the street, wishing to place the blame somewhere, calls the teacher inefficient—he knows that word by heart—and methods of teaching futile. But he knows, at least those running his schools for him know and tell him, that the technically efficient, the mechanically trained teacher is far cheaper, because far commoner, than the able individual teacher; and he would rather keep his tax rate low and grumble, than pay for education what it is worth and have no cause for complaint. He knows also, and wonders that it is so, that there are some teachers whose personality, whose character, fits them eminently for their profession, and he hopes that *his* children will fall to the lot of one of these.

III.

The second item named as a cause of the criticism of the teacher and his methods is the demand from the practical man, the man who runs a shop or a factory, for a child or a man who can do *his* work, and do it without further training; the demand that useful subjects be taught.

I for one believe, as I have already said, that the educator's first duty is to the cause of education; and this for the reason

that upon education depends the future of mankind. And with duty to his profession goes duty to the man and to the child. The teacher should, therefore, always bear in mind that it is the thinking human being whom he is training, not merely for his life's *work,* but for his life's entire activity. The practical man ignores this fact, or is indifferent to it. What he wants is an operative for his factory, a hand for his shop, a clerk for his office. That the clerk or the hand has a life outside of the factory or office he knows, of course, but to him it is of no moment. Nor does he seem to believe that education should in any way concern itself with this broader life of the family and of society.

This will be denied by many, and, doubtless, truthfully by some; but that it is the point of view of those who, as practical men, criticize the teacher and the educational system, we cannot doubt when we read such jeremiads as the following:

"We are still teaching our children to read fairy tales instead of watching moving pictures";—the writer says, but does not mean, that we teachers should be watching moving pictures instead of teaching our children to read fairy tales—"we teach them to write instead of training them on typewriters; we painfully drill into them multiplication tables instead of initiating them into the mysteries of the slide rule; we teach them to add and subtract instead of drilling them on comptometers; we teach them to draw instead of carefully training them to use photography; we have them drum for years on the piano even if they have no musical ability, when they ought to be trained to put a soul into mechanical records."

A child with "no musical ability" is to "put a soul" into the graphophone! I defy any Bradley Headstone or Miss Peecher of the old order to produce such a being as this system of training would produce.

Doubtless every child should be trained specifically for some trade or profession, but it does not follow that the school should give this training at the sacrifice of other more important things. If the school can educate the child for the broad and general activities of society; if it can help to form his character aright; if it can discipline him to obedience, trustworthiness, industry, accuracy; it will be doing all that can be demanded of the

school. The child's further training for some particular trade should be provided for, and paid for, by those demanding such training; or else it should be made possible for the child to obtain such training at the expense of the state, after he has received his general education.

The practical man will not consent to this, and as a resulting compromise we have, on the one hand, vocational schools which too often sacrifice essential general knowledge; and on the other hand, the introduction into the ordinary grammar and high school of what might be called frittering studies; "useful" studies of no disciplinary value in themselves, and positively harmful in the habit of mental laxity which they are permitted to encourage.

IV.

There are times in the history of a language when the people discover, or re-discover, a word: equality, evolution, organism, eugenics—the list might be made long, but we are concerned here only with the latest discovery, efficiency. Having discovered the word they proceed first to use, then to abuse, and finally to forget it. Efficiency seems to have reached the second stage. Now efficiency is a real thing and a thing to be desired; a thing to be sought with travail. But let us first know that which we seek. It can, presumably, be exactly defined; and if it is to be used as a basis of comparison between men, as it has long been between machines, it should be capable of measurement.

We will glance at one or two definitions of efficiency, offered by very practical engineers in the course of articles written for the guidance of the teacher.

One writer says efficiency is "the relation"—the context shows that he means ratio—"between what is and what could be." Presumably we must divide the actual teacher's "what is," by the ideal teacher's "what could be." As this writer in another place says that "what is" is probably wrong, and as we are not told how to obtain the "what could be," our rule for determining a teacher's efficiency would seem to be: Divide something which is incorrect by something else we don't know.

Another definition given is "the quotient of output divided by input"; where "output," the dividend, consists of "money

or salable goods, health, recreation, education, satisfaction"; and input, the divisor, consists of "time, money or raw material, physical labor, mental labor, nervous energy, health, wear and tear of machinery."

The writer says that in many cases . . . neither the whole output nor the whole input are [*sic*] capable of accurate measurement in similar terms." Yet we are to divide one by the other. And he adds, that if we take this definition "so as to include in the input every conceivable kind of expenditure and in the output every conceivable kind of achievement, it will apply to every activity of man." Why not of gods and men?

He says that the efficiency thus got "cannot be stated in figures"—not figures of speech— "as a percentage," and proceeds forthwith, in two typical cases, to get efficiencies of zero and one hundred per cent. respectively.

A third writer gives as a measure of the educator's usefulness

$$N \times H$$

where N is "the number of students taking a course," and H is "the average number of hours a week devoted by each in reciting under active criticism." He says, "this of necessity ignores the quality of the product"; as if the management of a shoe factory should count the number of pairs of boots made by a workman regardless of how badly they were made.

This $N \times H$ is "the numerator of a fraction expressing [the educator's] efficiency." The denominator is to be "estimated by fixing the maximum product of N and H possible with a reasonable expenditure of effort"!

Surely the teacher and the educational institution are justified in neglecting criticism and suggestions from such sources, unless they believe that they should

> "welcome each rebuff
> That turns earth's smoothness rough,
> Each sting that bids nor sit nor stand, but go!"

V.

Thus we see that the three causes of criticism of the teacher and of education walk hand in hand, form but one tendency of the times. We organize and systematize because it is easier and cheaper so to do; easier and cheaper to obtain technically "efficient" teachers than to obtain able teachers of individuality and character. Besides, it is such good form to say that our organization is scientifically managed, like *any other business*, and to declare that our teachers are proved efficient by actual computation. The practical man will be helpless when confronted by such statements, and will accept whatever "product" the school sends him.

I agree with one writer on "Academic Efficiency"—it is the only thing in his article with which I do agree—when he says, "It is high time that something practical be done in the way of reform" of the whole educational system. He adds, that such reforms "do not come about by normal process of evolution," and that "we therefore must look for a millionaire philanthropist to begin the great educational experiment."

In the opinion of the present speaker we can do much more than he says, and in a much better way. Let us retain just so much of our organization as will be helpful to the individual teacher and necessary for administration. Let us try no "great educational experiment" either with or without a captured millionaire. Let us rather simply allow the educational system to grow a natural, unforced growth, after we have pruned off all superfluous graftings, and have trimmed off all excrescences due to the stings of insects. Let us remember that the business of the educator is to educate; not, to be efficient.

One of our critics names, amongst other "principles of efficiency," "supernal common sense." Let me suggest that educators begin their reformation, to use Bacon's term, by adopting this principle. The consequence will be that the attempt to apply methods of industrial management to education will be abandoned. The attempt to measure the efficiency of a teacher by finding the ratio of two things, or sets of things, neither of which can be measured, or by neglecting the very essence of education, will be abandoned. The attempt to satisfy the un-

reasonable demands of the practical man at his own pitiful price, will be abandoned. The sacrifice of the higher aims of education to material gain, will be abandoned.

There is a story that when Zola's remains were being transferred to the Panthéon, an anti-Semite fired a shòt at Dreyfus, who was present. When tried, the defendant declared that he had no hatred for Dreyfus; had not even aimed at him. "My gesture was symbolic, and I fired the shot at an idea," he said.

The present speaker, also, is an idealist. He has no hatred for any efficient teacher or superintendent, and has not even aimed at any. He has simply fired his shot at an idea.

Massachusetts Institute of Technology,
Boston, Mass.

NEW BOOKS.

The Art of the Wallace Collection. By HENRY C. SHELLEY. Boston: L. C. Page and Company. Pp. 334. $2.00 net.

The publishers of this book have done a good service in bringing out so many books on the art collections of Europe. They give those who cannot visit the galleries a good idea of their content and even for those who have or expect to visit the galleries the reading of these books would add much to their interest and appreciation.

The Wallace collection has been open to the public less than thirteen years but in that time has become very popular and is held in the highest esteem.

The author gives his own estimate of the various pieces and has produced what would seem a valuable account of this important collection, which includes besides pictures, furniture, bronzes, arms, armour, etc. The book is well illustrated and presents a fine appearance.

The Russian Empire of To-Day and Yesterday. By NEVIN O. WINTER. Boston: L. C. Page and Company. Pp. 487. $3.00 net.

The least known to Americans of all the great countries of Europe is undoubtedly that of Russia. This is due to the fact that American travelers do not visit it as they have other European countries, and also to the fact that much that has been written about it has a bias.

The author of this volume has done a service in giving a careful account of the country, its people and something of its history past and present. He also gives a survey of social, political and economic conditions of the country. It is beautifully gotten up and well illustrated by photographs taken by the author.

Pussy Black-Face. By MARSHALL SAUNDERS. Boston: L. C. Page and Company. Pp. 311. $1.50.

The author has in this work given in an interesting way the story of a kitten and her friends. Such stories will do much to teach children a needed lesson in the treatment of dumb animals and especially cats.

Pollyanna. By ELENOR H. PORTER. Boston: L. C. Page and Company. Pp. 310. $1.25 net.

This book, known as the "Gladd Book," is a story of how a little girl of eleven transformed a whole community by playing the game of *glad* which was to always find something in every event to be glad about. It is a splendid story well told.

Our Little Bulgarian Cousin. By CLARA VOSTROVSKY WINLOW. Boston: L. C. Page and Company. Pp. 115. 60 cents.

This is one of the "Little Cousin Stories" which has been so popular for young people. It is well written and gives the reader a good insight into Bulgarian life and conditions of to-day.

Our Little Roman Cousin of Long Ago. By JULIA DARROW COWLES. Boston: L. C. Page and Company. Pp. 118. 60 cents.

This is the initial volume of a new series, companion to that of the "Little Cousin Series," which is intended to give in an interesting way an accurate account of the life and times of the people of long ago. The period chosen for this volume is the final period of the Republic, the most dramatic in all Roman history, and such personages as Cæsar, Cataline, Cicero, and Pompey figure in its pages.

The author and publishers deserve credit for producing such interesting and profitable books for young people as is found in these series.

University and Historical Addresses. By JAMES BRYCE. New York: The Macmillan Company. Pp. 443. $2.25 net.

Mr. Bryce's wide experience and profound scholarship are such as to make his writings of more than ordinary interest, and the present volume is no exception to that rule. His thoughts on educational topics should be of particular interest as they possess a balance too often lacking in this country. Among the topics treated are: What University Instruction may do to Provide Intellectual Pleasure for Later Life; The Mission of the State Universities; What a University may do for a State; Special and General Education in Universities; The Study of Ancient Literature; Architecture and History; Some Hints on Public Speaking; Some Hints on Public Reading; The Character and Career of Abraham Lincoln; etc. His clear thought stated in precise language makes them all delightful as well as profitable reading.

Trigonometry. By ALFRED MONROE KENYON and LOUIS INGOLD. New York: The Macmillan Company. Pp. 256. $1.35 net.

This book covers the usual course in trigonometry and contains quite a comprehensive set of tables. It begins with the right triangle and devotes some attention to graphs. The solution of triangles is the principal motive, and all pure theoretical work which does not bear on this is omitted.

A First Course in Algebra. By FREDERICK C. KENT. New York: Longmans, Green and Company. Pp. 262. $0.00.

The author had two objects in view in the preparation of this book, viz., to give the student who will soon drop out of school a sufficient course to serve him in elementary science, in business, or in industrial life; and to give those who do go on a good foundation for their future work. The equation is made the central idea and much attention is given to the solution of problems.

School Algebra. Book I. By GEORGE WENTWORTH and DAVID EUGENE SMITH. Boston: Ginn and Company. Pp. 298. 90 cents.

The "School Algebra," which is a somewhat extended treatment of the material in the author's "Academic Algebra," is admirably adapted

for those teachers who prefer a two-book arrangement, and offers ample material for a full two years' course. Book I covers algebra through an elementary treatment of quadratics, and provides a chapter on ratio and proportion that may be taken before geometry is begun. Book II contains a thorough review of Book I, with new and somewhat more difficult problems, gives a more extended treatment of quadratics, and carries the work through progressions, the Binomial Theorem, and complex numbers. Book I may easily be abridged to meet the needs of classes that are not prepared to undertake the work in quadratics. Although Book II provides a full year's course, the essential features may be covered in a half year by omitting the review and the chapter on Complex Numbers.

Teaching of Arithmetic. By DAVID EUGENE SMITH. Boston: Ginn and Company. Pp. 196. $1.00.

This work on the teaching of arithmetic has been prepared to meet the needs of reading circles and of teachers in the elementary school. It considers the origin of arithmetic, the reasons for teaching the subject, the various noteworthy methods that have been suggested, and the work of the various school years. There is also a discussion of the subjects to be included, the nature of the problems, the arrangement of material, the place of oral arithmetic, the nature of written arithmetic, the analyses to be expected of children, the modern improvements in the technique of the subject, the question of interest and effort, the proper subjects for experiment, and the game element that plays such an important part in the primary grades.

The book is not one of explanations of mathematical processes, nor is it concerned with little devices, these being sufficiently developed in the ordinary textbooks or so generally known as to make it unnecessary to dwell upon them. On the other hand, it seeks to set before the teacher the larger phases of the subject and to encourage her to progressive, enthusiastic, intelligent work in the grades of the elementary school. It is written in a non-technical style that will appeal to all readers, and it supplies a large bibliography that will help teachers who wish to investigate the subject further.

The Theory of Relativity. By ROBERT D. CARMICHAEL. New York: John Wiley & Sons. Pp. 74. $1.00.

This is No. 12, of the series of Mathematical Monographs edited by Mansfield Merriman and Robert S. Woodward. The subject is one which has considerable interest though many physicists seem to pay little attention to it. For those who desire an introduction to the theory this book will prove very useful as it is written in such a way as to make easy reading. Among others the following topics are treated: The Postulates of Relativity; The Measurement of Length and Time; Equations of Transformation; Experimental Verification of the Theory.

NOTES AND NEWS.

THE spring meeting of the Philadelphia Section of the Association of Teachers of Mathematics in the Middle States and Maryland was held in the Central High School, Philadelphia, on Wednesday afternoon, April 16, 1913, President Maurice J. Babb in the chair.

Henry G. Gummere, professor of mathematics, Drexel Institute, presented an interesting paper on "The Reduction of Observations."

The Discussion, "Should Algebra be Taught in the Public Secondary Schools?" was led by Jesse D. Burks, head of the Bureau of Municipal Research of Philadelphia; George Gailey Chambers, Chairman of the Committee on Advanced Standing, University of Pennsylvania; William D. Lewis, principal of the William Penn High School for Girls, and J. Eugene Baker, principal of the Philadelphia High School for Girls. They were followed by remarks by mathematicians, scientists and others—all on the affirmative side of the question.

The following officers were elected for the ensuing year:—*President,* Dr. Fletcher Durell, Laurenceville School, Laurenceville, N. J.; *Vice-President,* Mr. Thomas Moore, West Philadelphia High School for Boys; *Secretary,* Elizabeth B. Albrecht, Philadelphia High School for Girls; *Members of the Executive Committee,* Mrs. Katharine D. Brown, Drexel Institute, Professor George G. Chambers, University of Pennsylvania.

Free to Teachers.—Any teacher, upon request, will receive without expense a copy of a new booklet, "Jack." This little story, copyrighted by Dr. Charles A. Coulomb, Ph.D., contains interesting and helpful suggestions on class drill in the use of a dictionary. Why not make use of "Jack's" experiences to teach your pupils the advantages of early forming the dictionary habit? Address the publishers, G. & C. Merriam Co., Springfield, Mass.

With the opening of school our Association should take on

new life and helpfulness. Every member should try and enlist the interest of at least one who is not a member and get them to join.

The Free Spirit of Modern Progress.—For the information of many inquiring friends, it seems wise at this time to say that there will be no "new" *Century* in the sense of a changed *Century*. There can be none. In remaining the "old" *Century,* merely growing with the times, merely holding fast to its historic place in the front of progress, this magazine, in these richer days of hard thinking and prompt acting and strenuous living, these tumultuous days of changing eras, remains by mere definition the organ of what is noblest and forwardest in American life. The first editor of this magazine stated editorially that it was conducted in "the free spirit of modern progress and the broadest literary catholicity." The fourth editor joyfully reaffirms this creed. There can be no simpler and more comprehensive statement of this magazine's present spirit and purposes.

To Promote Good Citizenship.—In the twentieth-anniversary number, Richard Watson Gilder, who, on Dr. Holland's death in 1881, succeeded to the editorship, reaffirmed the creed in these words: "If there is any one dominant sentiment which an unprejudiced reviewer would recognize as pervading these forty half-yearly volumes, it is, we think, a sane and earnest Americanism. Along with and part of the American spirit has been the earnest endeavor to do all that such a publication might do to increase the sentiment of union throughout our diverse sisterhood of States—the sentiment of American nationality. It has always been the aim of *The Century* not only to be a force in literature and art, but to take a wholesome part in the discussion of great questions; not only to promote good literature and good art, but good citizenship." Allowing for different conditions, Mr. Gilder might have written this for to-day. In the same editorial utterance Mr. Gilder dwelt strongly upon "the spirit of experiment" which, he said, had always inspired *The Century's* policy. This we take to be merely another phrase for Dr. Holland's "free spirit of modern progress." Five years later, on the occasion of *The Century's* twenty-fifth anniversary, Mr. Gilder wrote in these pages: "During the next

ten years there should be in America especially a revival of creative literature. If there is, or should be at any particular time, a lack of energy, or a lack of quantity or quality, in the American literary output, it can be merely temporary; for our condition is full of social, political, and industrial problems; life in the New World is replete with strenuous exertion of every kind, of picturesque contrasts, and of innumerable themes fit to inspire literary art. American life is rich in feeling and action and meaning."